Intrinsic localized modes in microelectromechanical oscillator arrays: a dynamical approach

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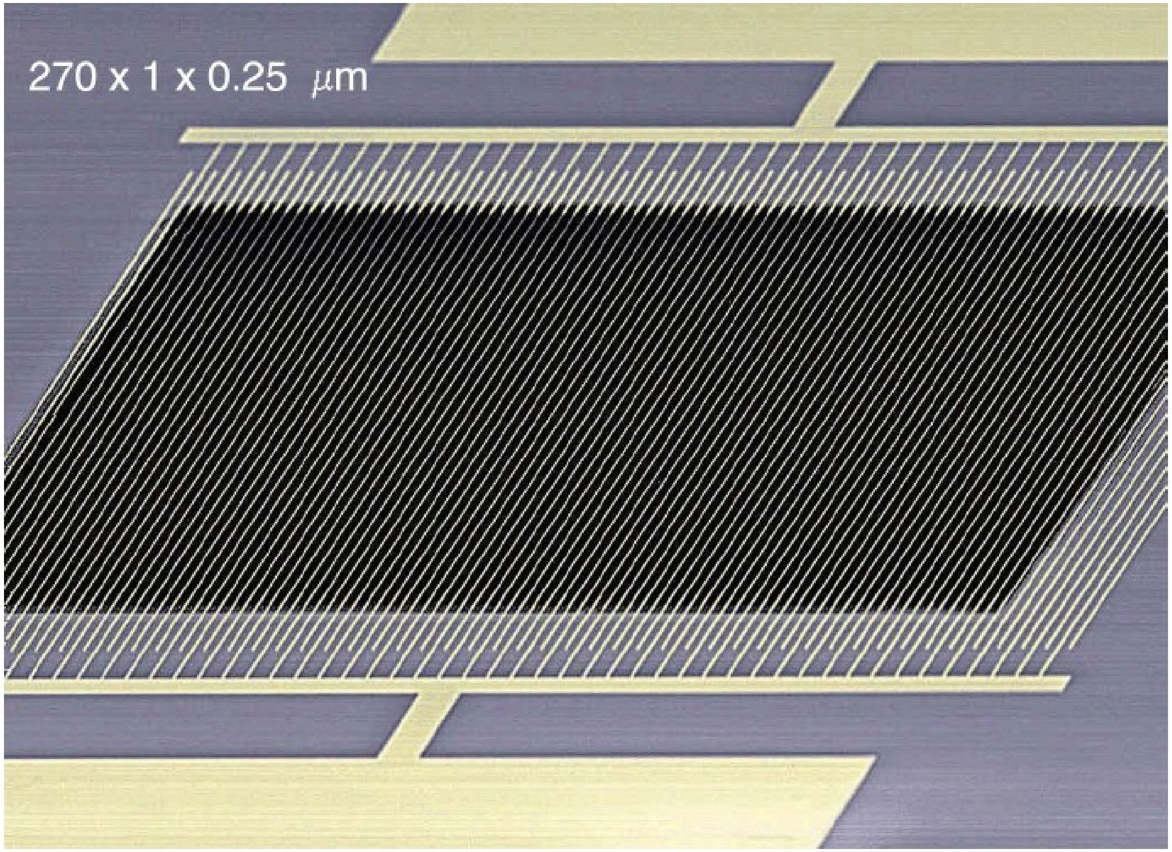
Microelectromechanical (MEM) arrays

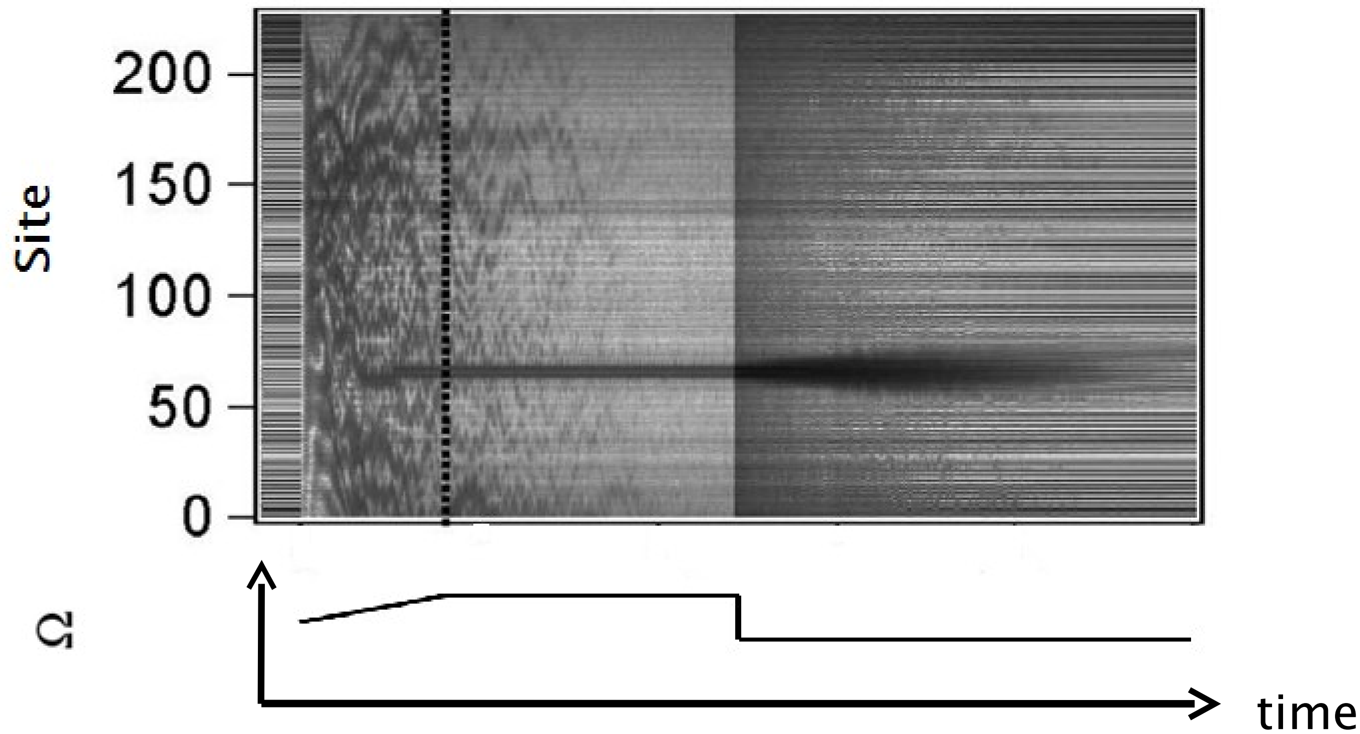
MEMS device in array structure:



- Micro resonator
- Micro sensor
- Micro mechanical filter
- Micro generator
- ●
- ●
- ●

E. Buks and M. L. Roukes, J. Microelectromech. Syst., 11, 802 (2002).

Intrinsic localized modes (ILMs) in MEM oscillator array: Experiment



M. Sato et. al., Rev. Mod. Phys. 78, 137 (2006).

- The dynamical mechanism for ILMs in MEMS was not very clear.

↓

- We propose a dynamical analysis for better understanding of ILMs' creation mechanism.

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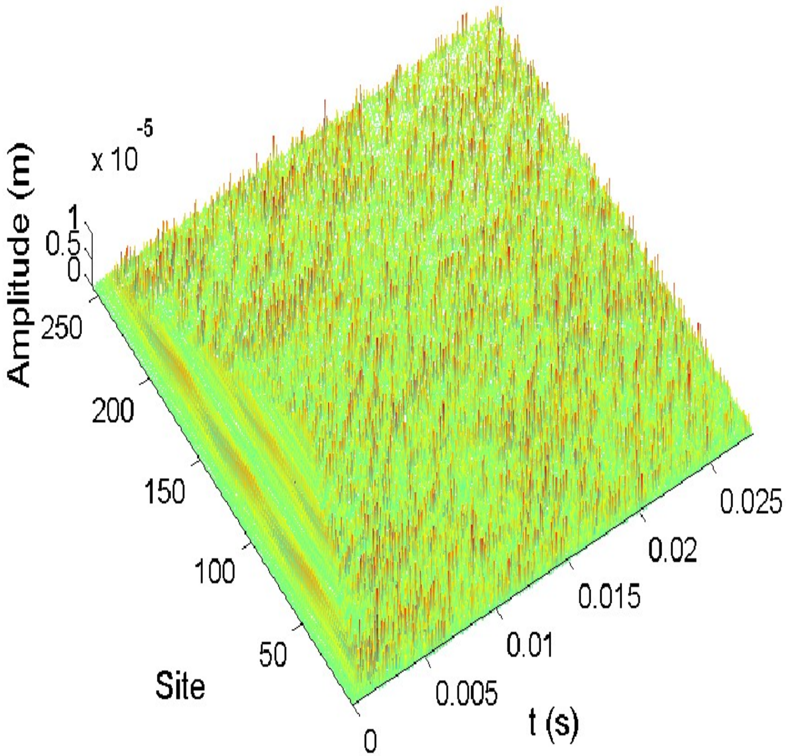
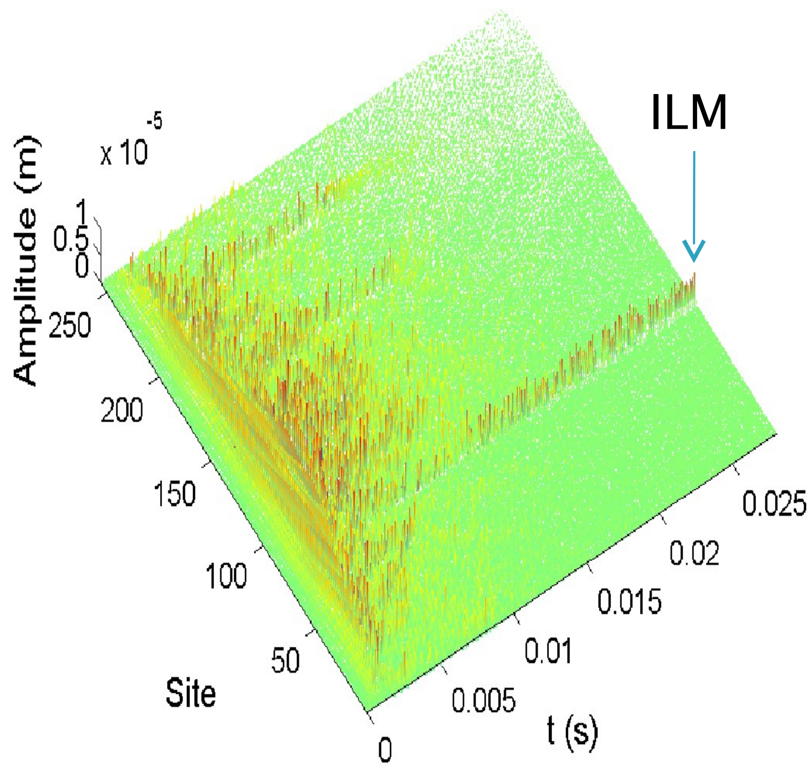
- We find that ILMs can be excited from inherent spatiotemporal chaos.

Spatiotemporal chaos and intrinsic localized modes in MEM arrays

Inherent spatiotemporal chaos:

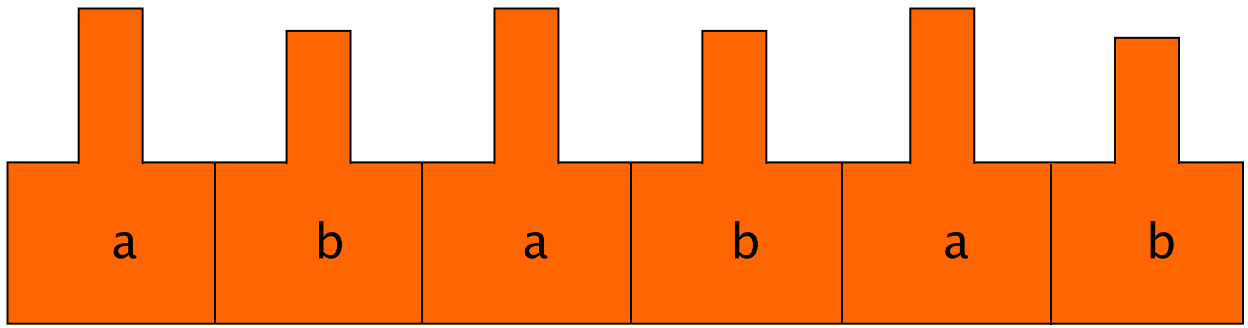


Intrinsic localized mode:



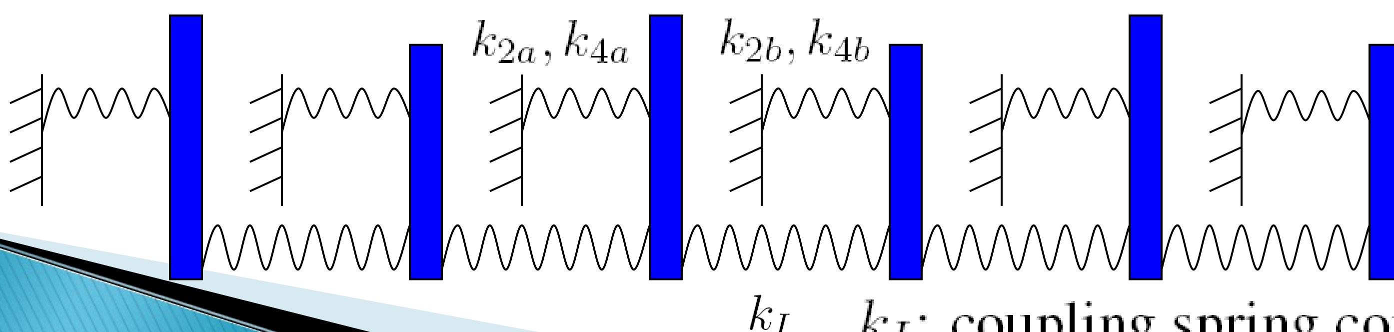
Experimental settings

Cantilever array structure and its equivalent mass-spring model:



k_{2a}, k_{4a}: harmonic and anharmonic spring constants of long beams

k_{2b}, k_{4b}: harmonic and anharmonic spring constants of short beams



k_I: coupling spring constants

Dynamic model of MEM arrays

MEM cantilever arrays equation:

$$m_i\ddot{x}_i + b_i\dot{x}_i + k_{2i}x_i + k_{4i}x_i^3 + k_I(2x_i - x_{i+1} - x_{i-1}) = m_i\alpha\ \cos(\Omega\ t),$$

where $x_i(i = 1,2,\ldots,N)$ N is even

$$(m_i, b_i, k_{2i}, k_{4i}) = (m_a, b_a, k_{2a}, k_{4a})$$

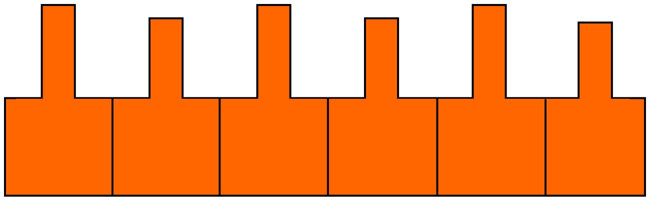
$= (5.5\times 10^{-13}\text{kg}, 6.2\times 10^{-11}\text{kg/s}, 0.303\text{N/m}, 5\times 10^{8}\text{N/m}^3)$ if i is odd

$$(m_i, b_i, k_{2i}, k_{4i}) = (m_b, b_b, k_{2b}, k_{4b})$$

$= (5.0\times 10^{-13}\text{kg}, 5.7\times 10^{-11}\text{kg/s}, 0.353\text{N/m}, 5\times 10^{8}\text{N/m}^3)$ if i is even

m_i : mass

b_i : damping coefficient



Averaged model for MEM arrays

$$x_i(t) = U_i(t)\cos(\Omega t) + V_i(t)\sin(\Omega t)$$

$$\frac{du_i}{dt} = -\frac{1}{2\Omega}[(\Omega^2 - \Omega_{0i}^2)v_i - \frac{3k_{4i}}{4m_i}v_i(u_i^2 + v_i^2) + \frac{\Omega_{oi}}{Q_i}\Omega u_i - \frac{k_I}{m_i}(2v_i - v_{i+1} - v_{i-1})],$$

$$\frac{dv_i}{dt} = -\frac{1}{2\Omega}[(\Omega^2 - \Omega_{0i}^2)u_i - \frac{3k_{4i}}{4m_i}u_i(u_i^2 + v_i^2) - \frac{\Omega_{oi}}{Q_i}\Omega v_i + \alpha - \frac{k_I}{m_i}(2u_i - u_{i+1} - u_{i-1})],$$

where $\Omega_{oi} = \sqrt{k_{2i}/m_i}, Q_i = \sqrt{m_i k_{2i}}/b_i$

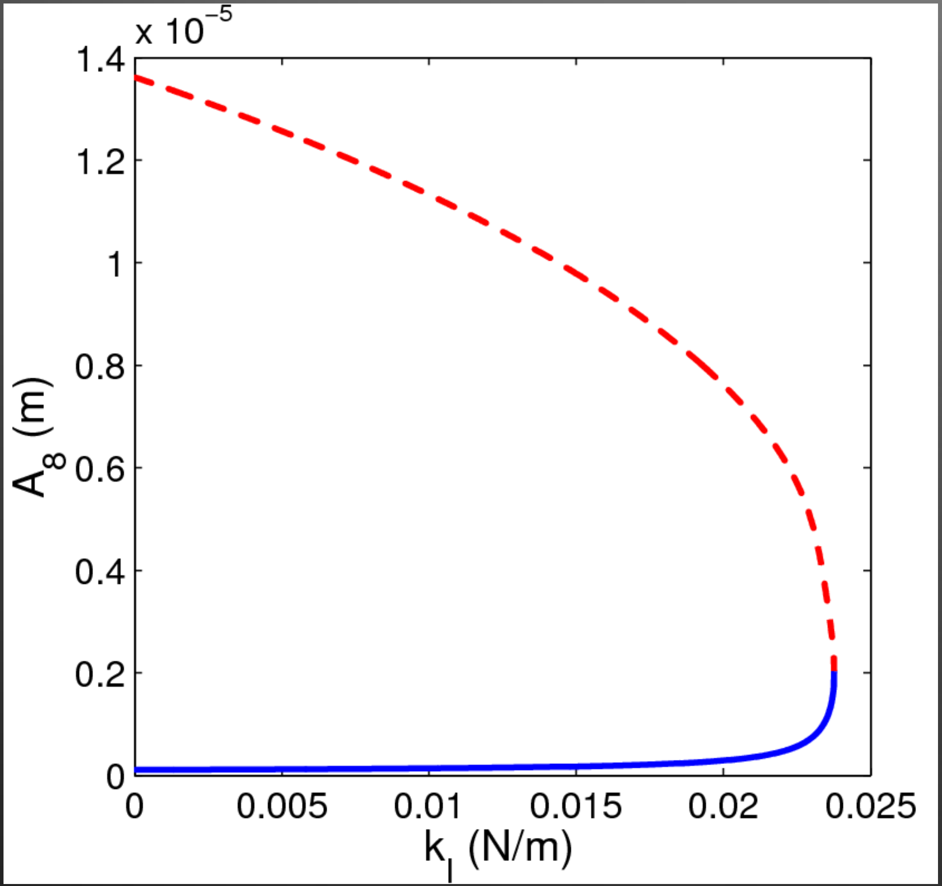
u_i and v_i are the average functions of U_i and V_i over one period.

Amplitudes and phase angles can be approximated by:

$$A_i = \sqrt{u_i^2 + v_i^2}, \theta_i = \arctan(v_i/u_i)$$

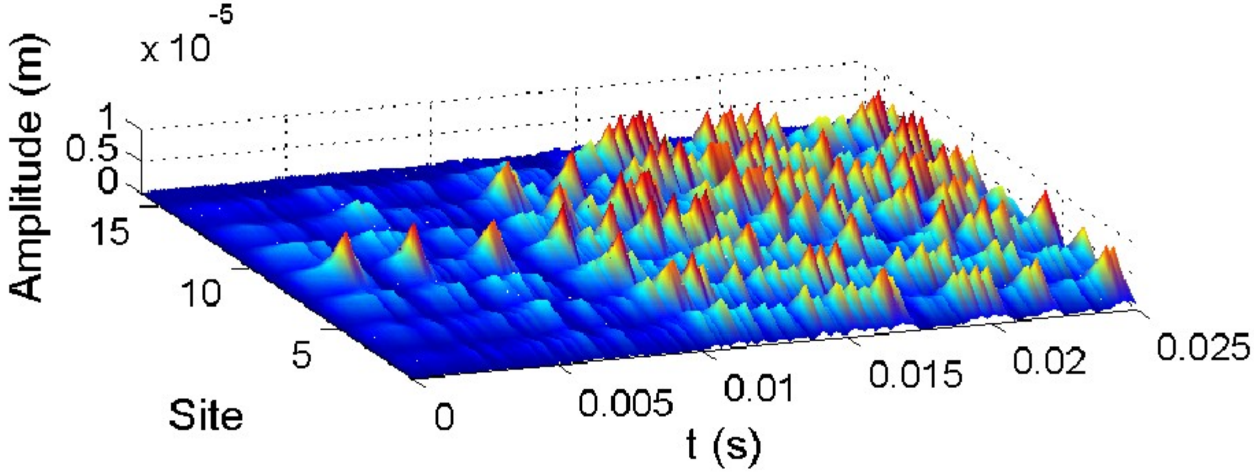
Saddle-node Bifurcation to spatiotemporal chaos

Bifurcation diagram of low energy state in averaged system (N=16):

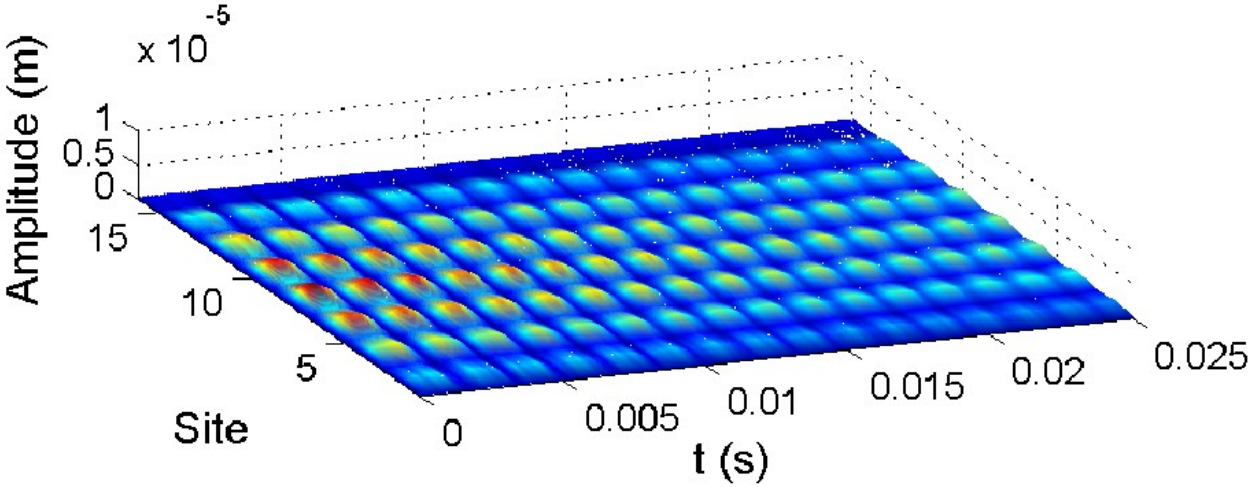


Saddle-node bifurcation to spatiotemporal chaos

$k_I = 0.0236\,\mathrm{N/m}$



$k_I = 0.0235\,\mathrm{N/m}$



Generating ILMs from spatiotemporal (ST) chaos

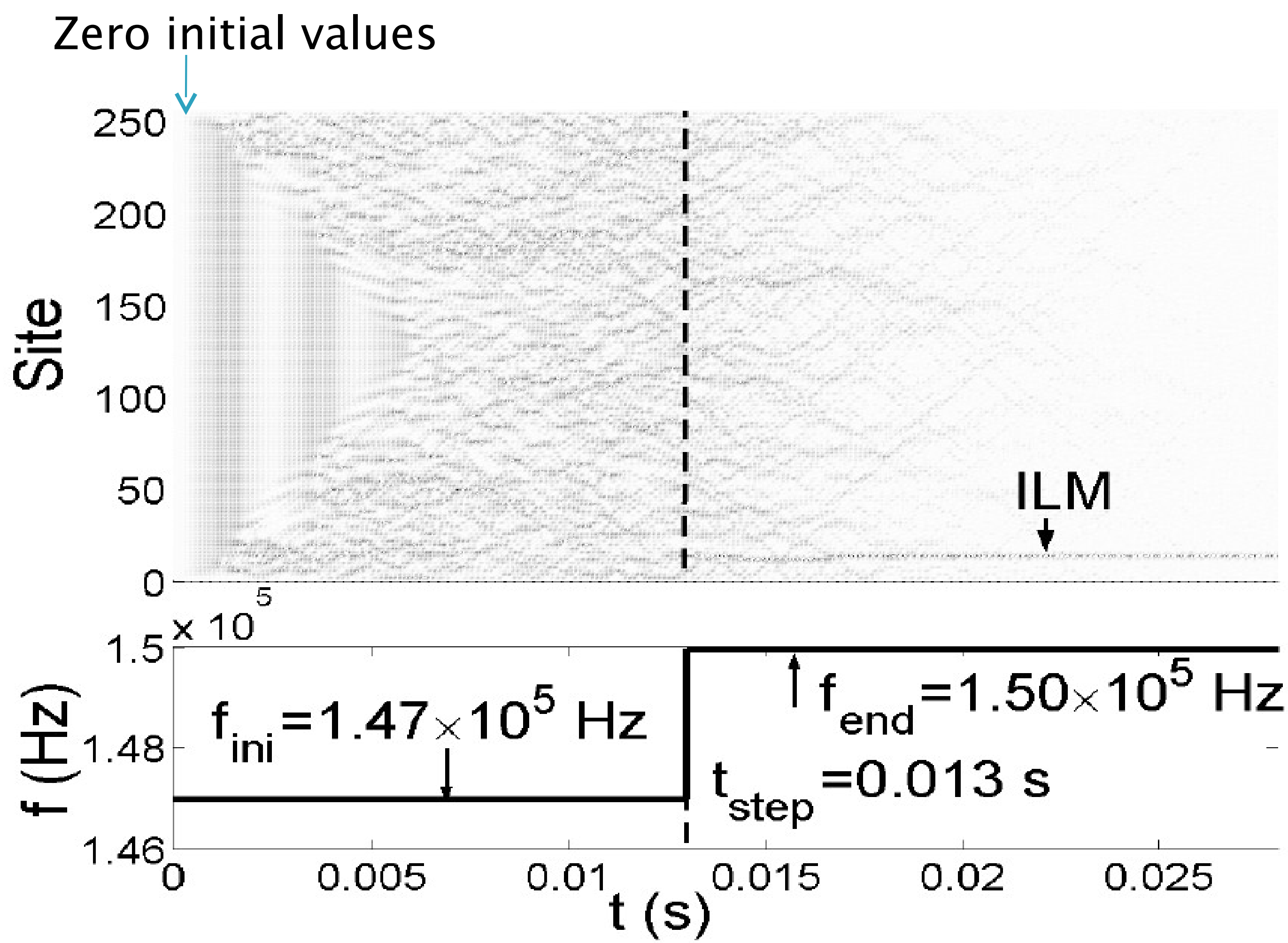
In the previous literature, people considered that

1 Initial noise
2 Frequency chirping scheme

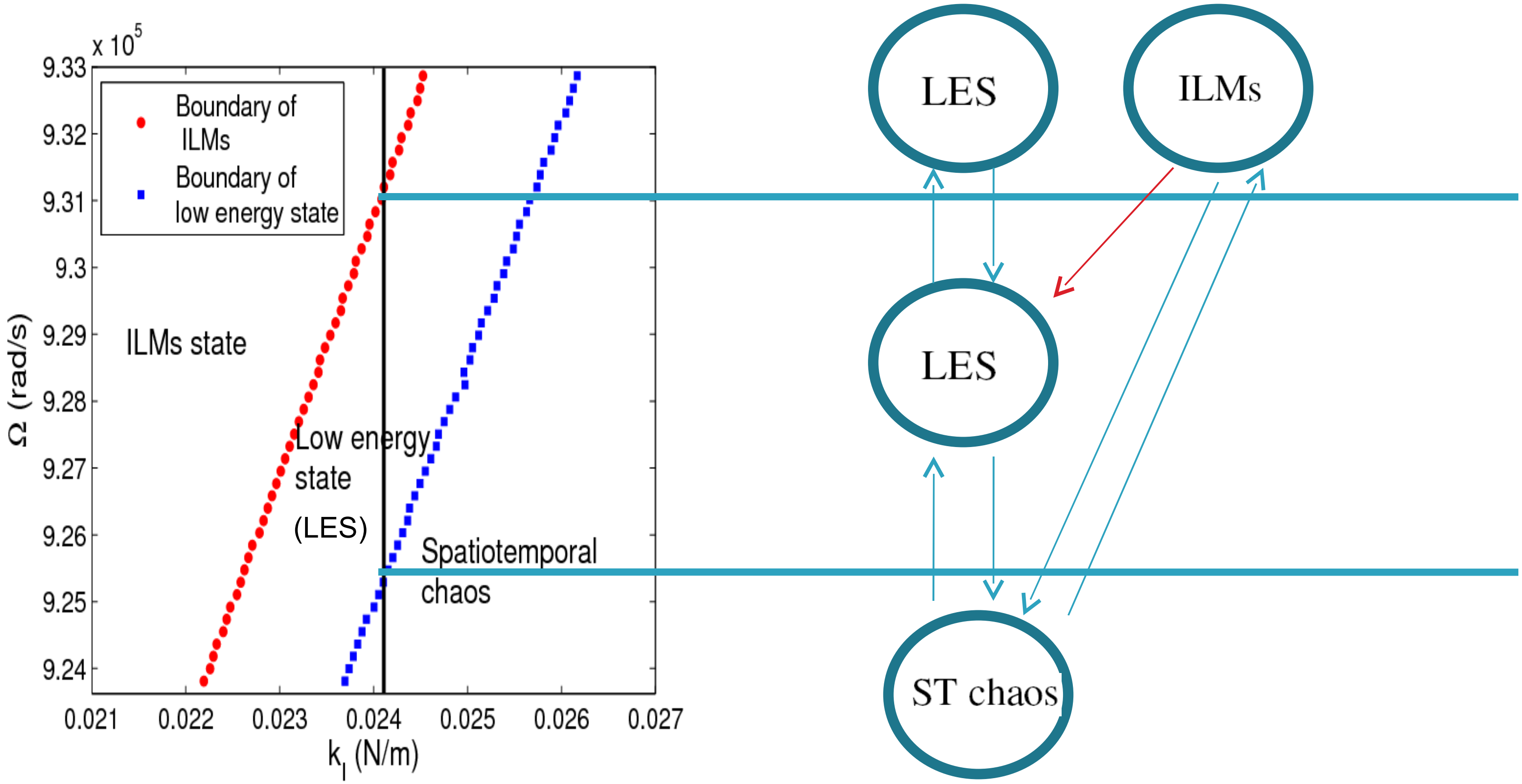
are necessary for generating ILMs in MEM arrays.

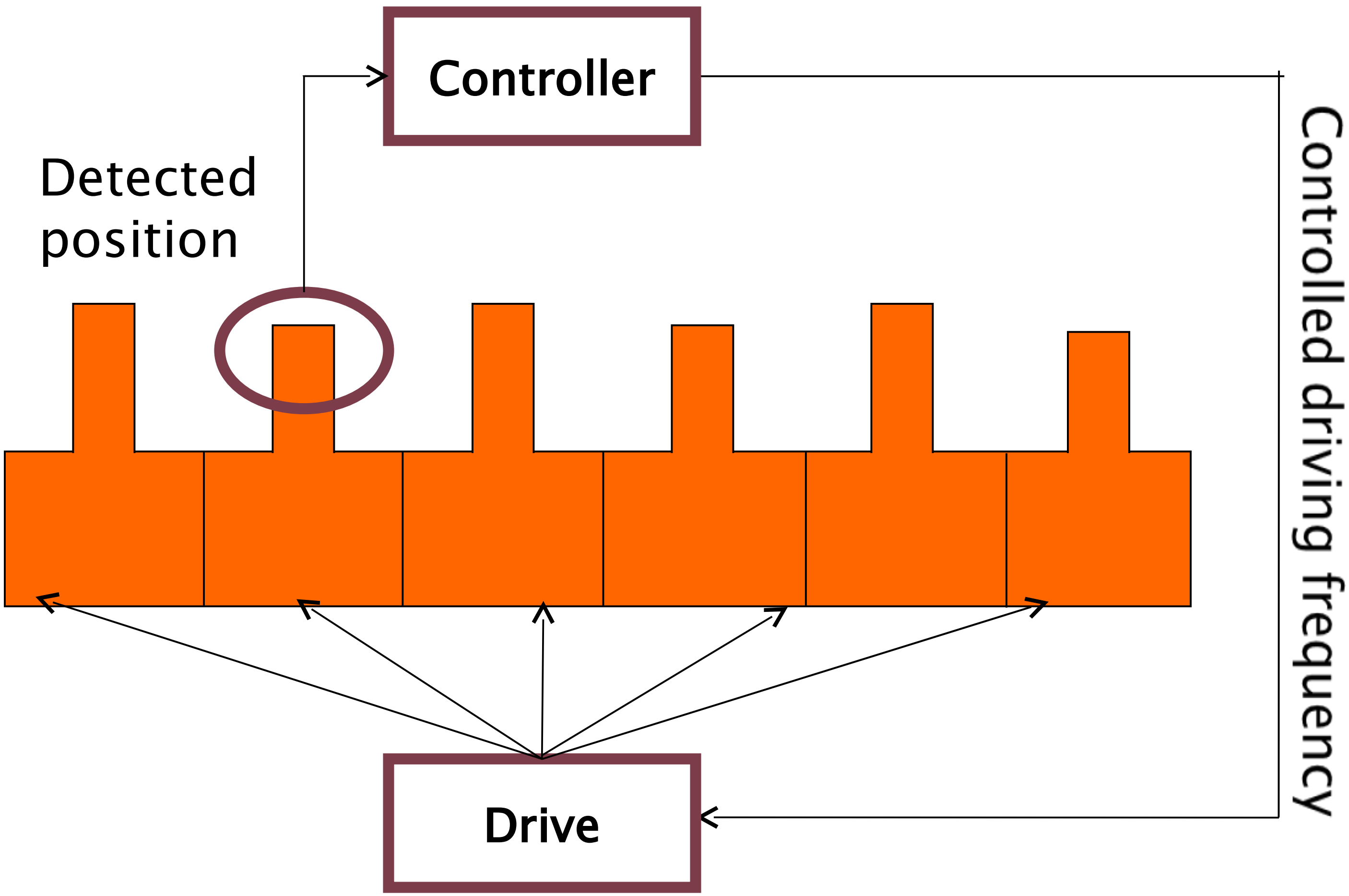
However, due to inherent ST chaos, these conditions can be relaxed.

Creating ILMs from chaos

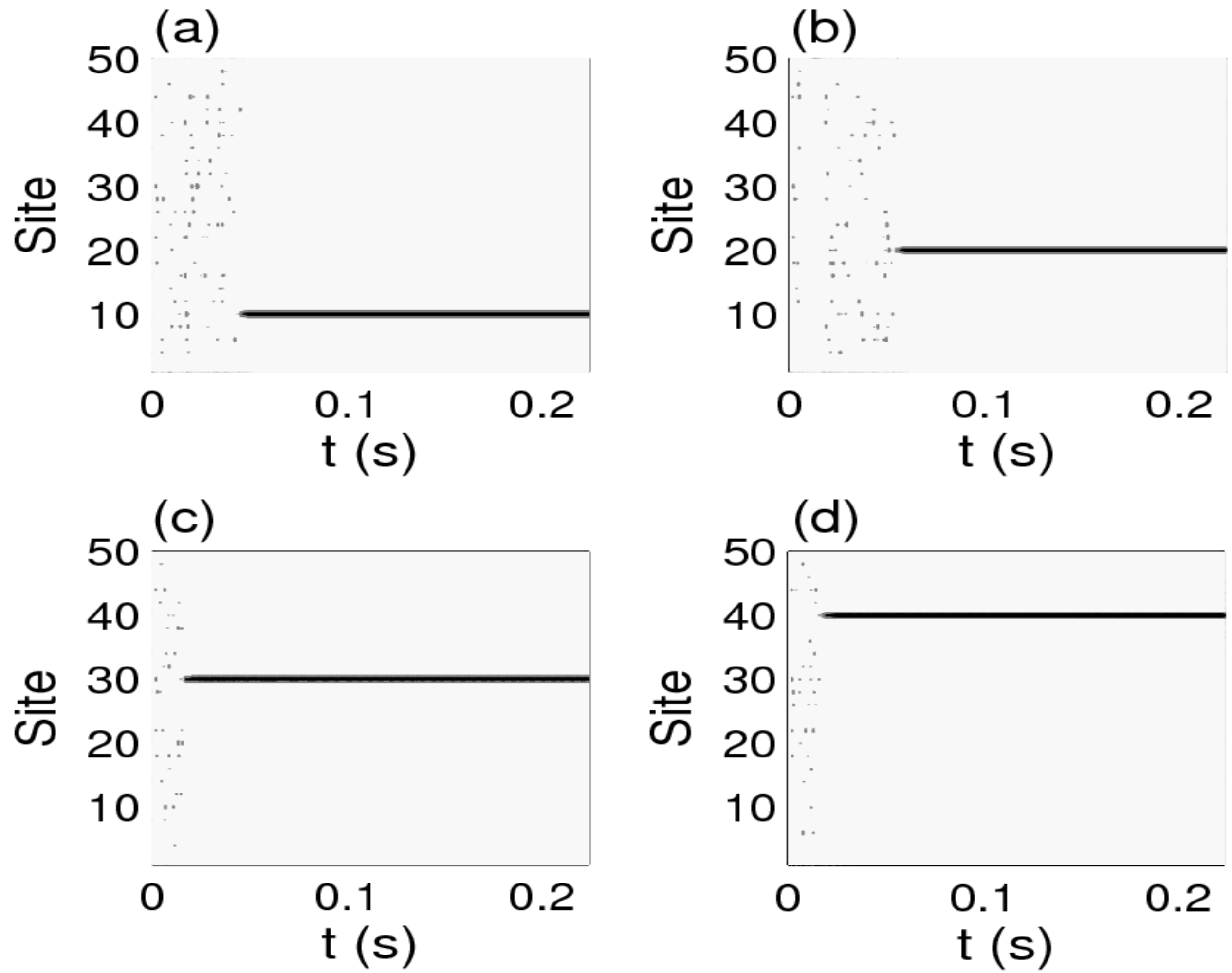


Creating ILMs from ST chaos




Controller
Detected position
Controlled driving frequency
Drive

Different patterns in MEM array

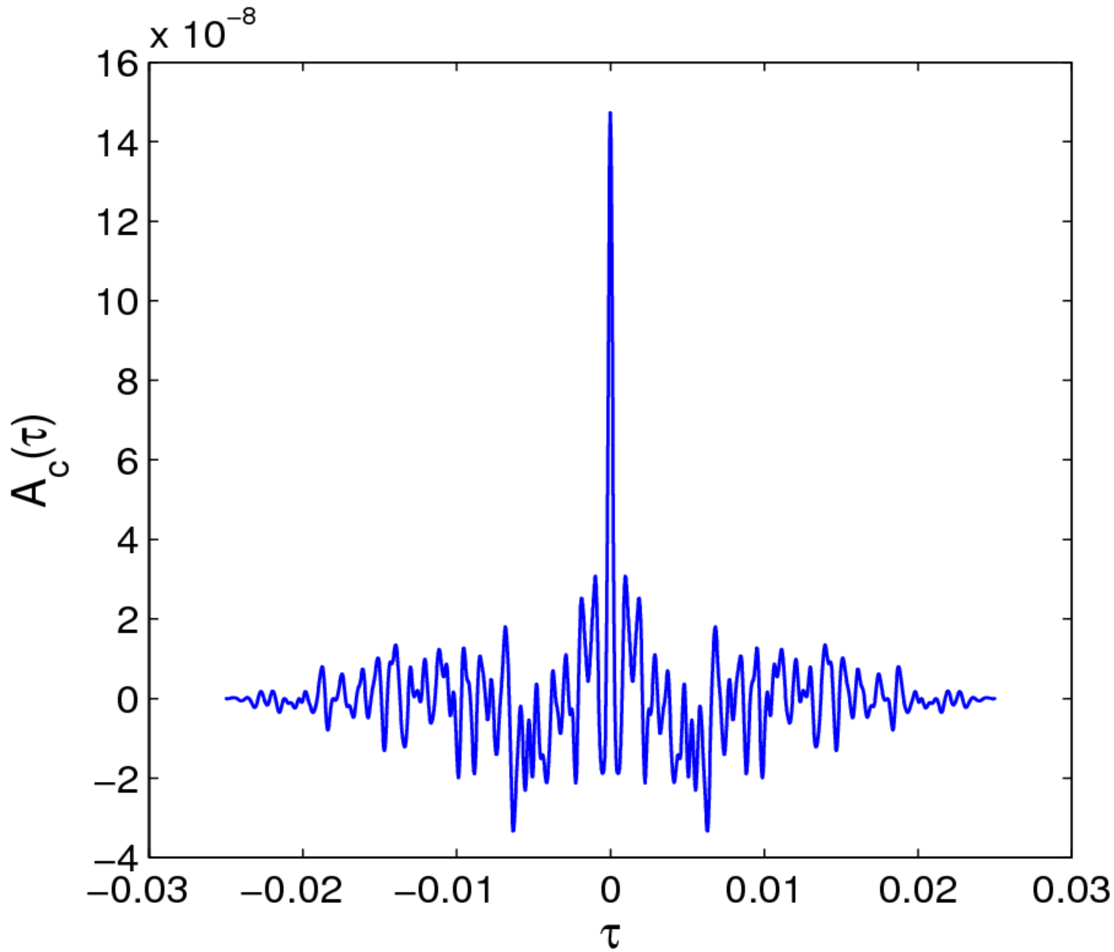


Conclusion

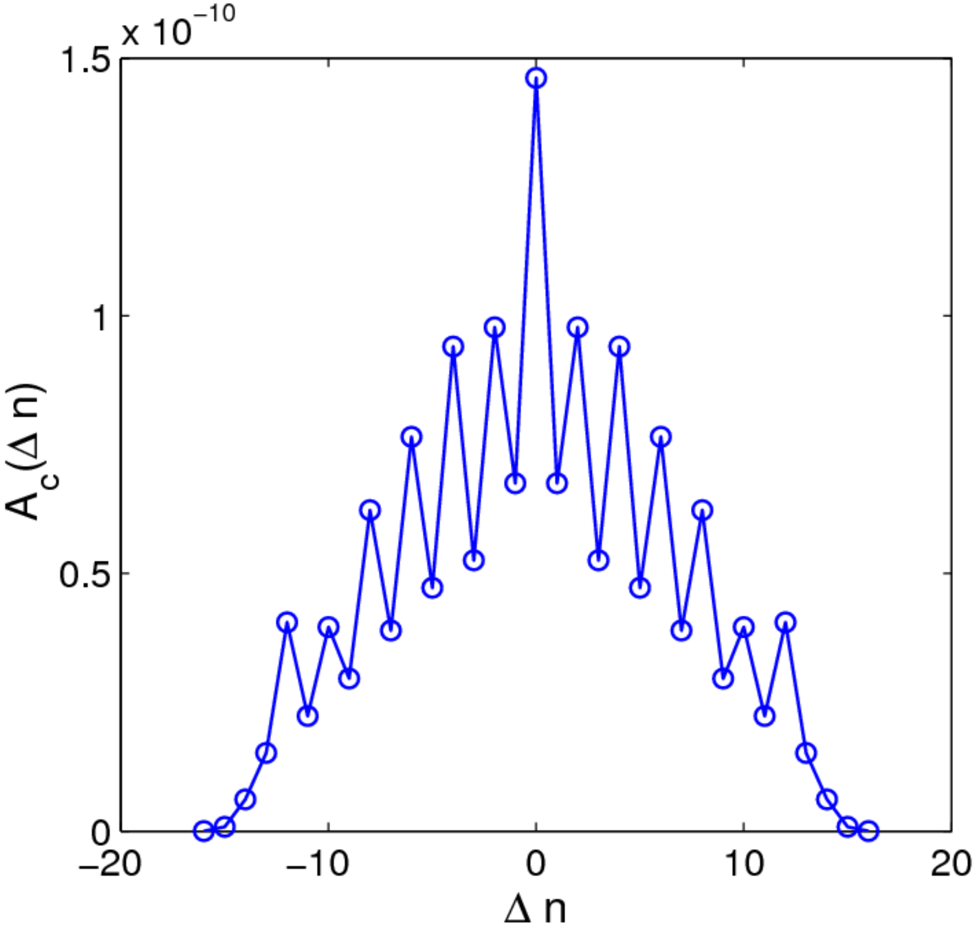
- An averaged model to study the dynamics of driven mircocantilever arrays.
- Spatiotemporal chaos serves as a natural exciting platform for ILMs.
- Forming different patterns of ILMs in MEM cantilever arrays.

Thanks!

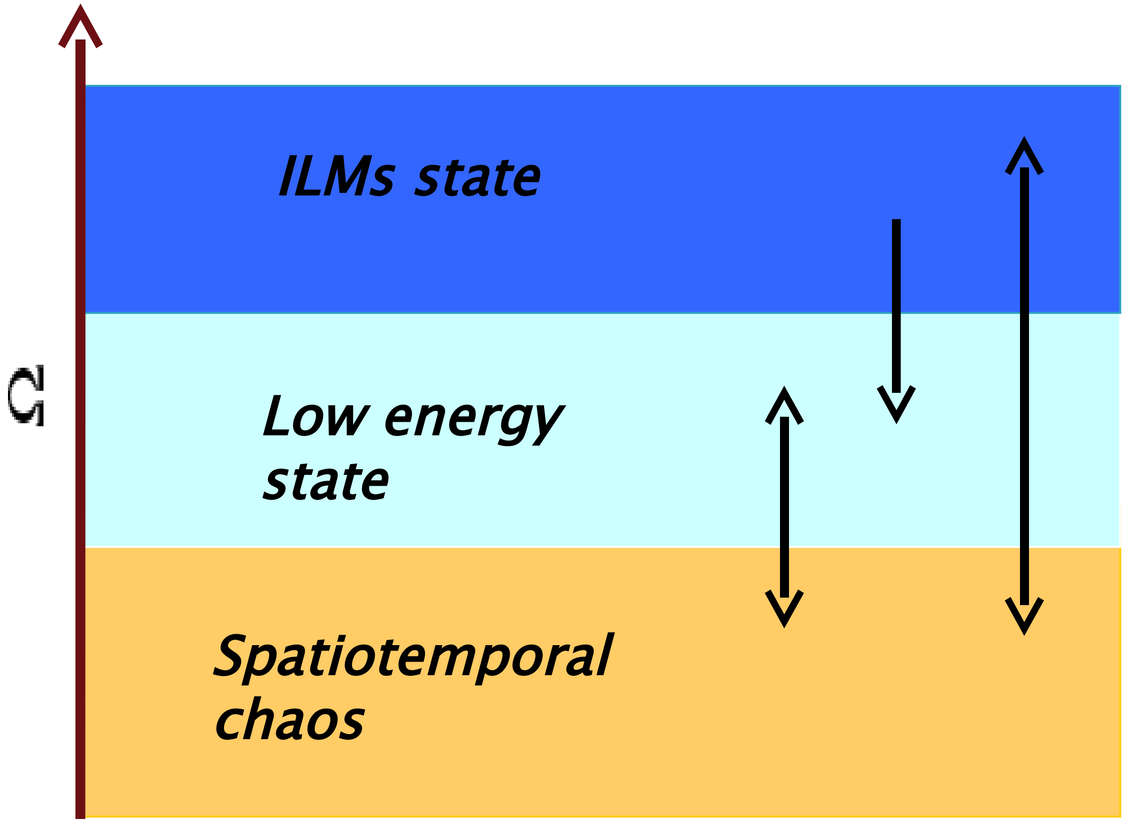
Correlation function of time



Correlation function in space



Creating ILMs from chaos



Controlled MEM arrays

$m_i\ddot{x}_i + b_i\dot{x}_i + k_{2i}x_i + k_{4i}x_i^3 + k_I(2x_i - x_{i+1} - x_{i-1}) =$
$m_i\alpha\cos(\Omega_c t)$, where x_i $(i = 1, \ldots, N)$

$$\begin{cases} \dot{\Omega}_c = \gamma\xi \\ \dot{\xi} = -(1/\tau)(\xi + x_M \sin(\Omega_c t + \phi)) \end{cases}$$

Dynamic model of MEM arrays

MEM cantilever arrays equation:

$$m_i\ddot{x}_i + b_i\dot{x}_i + k_{2i}x_i + k_{4i}x_i^3 + k_I(2x_i - x_{i+1} - x_{i-1}) = m_i\alpha\cos(2\pi f t),$$

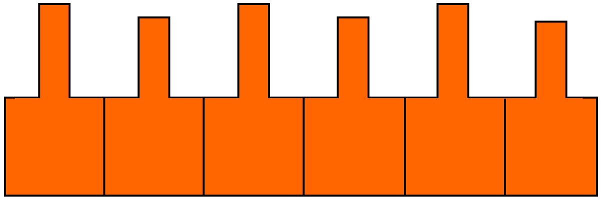
where x_i $(i = 1, 2 \ldots N)$ N is even

$$(m_i, b_i, k_{2i}, k_{4i}) = (m_a, b_a, k_{2a}, k_{4a}) \quad \text{if } i \text{ is odd}$$

$$(m_i, b_i, k_{2i}, k_{4i}) = (m_b, b_b, k_{2b}, k_{4b}) \quad \text{if } i \text{ is even}$$

m_i : mass

b_i : damping coefficient



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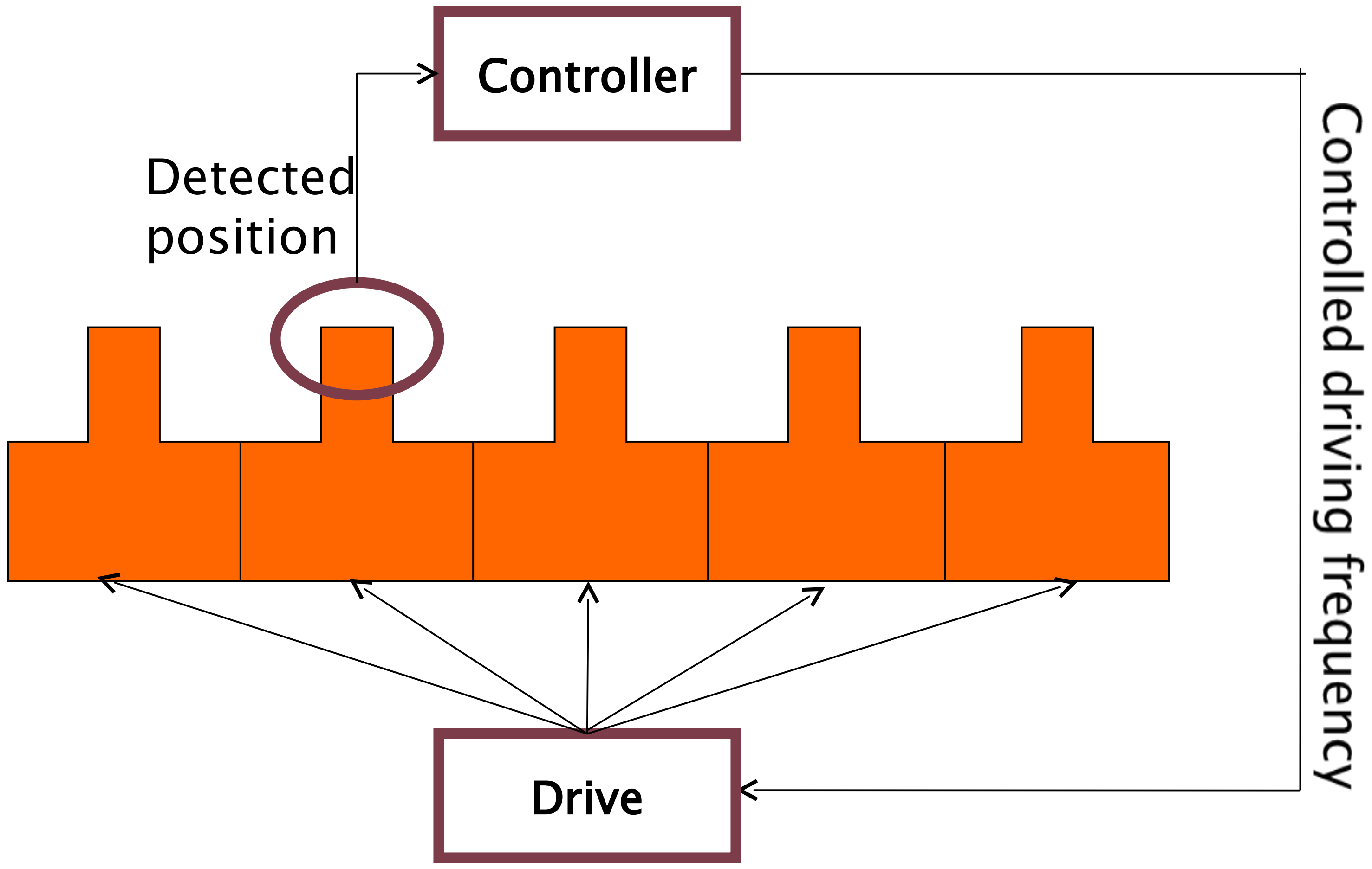
$$\begin{aligned}\frac{du_i}{dt} &= -\frac{1}{2\Omega}[(\Omega^2 - \Omega_{0i}^2)v_i - \frac{3}{4}\frac{k_{4i}}{m_i}v_i(u_i^2 + v_i^2) + \frac{\Omega_{0i}}{Q_i}\Omega u_i \\ &\quad -\frac{k_I}{m_i}(2v_i - v_{i+1} - v_{i-1})], \\ \frac{dv_i}{dt} &= \frac{1}{2\Omega}[(\Omega^2 - \Omega_{0i}^2)u_i - \frac{3}{4}\frac{k_{4i}}{m_i}u_i(u_i^2 + v_i^2) - \frac{\Omega_{0i}}{Q_i}\Omega v_i \\ &\quad +\alpha - \frac{k_I}{m_i}(2u_i - u_{i+1} - u_{i-1})], \quad i = 1, \ldots, N.\end{aligned}$$

where $\Omega_{0i} = \sqrt{k_{2i}/m_i}$, $Q_i = \sqrt{m_i k_{2i}}/b_i$

u_i and v_i are the averaged functions of $U_i(t)$ and $V_i(t)$ over one period, respectively.

Amplitudes and phase angles can be approximated by:

$$A_i = \sqrt{u_i^2 + v_i^2}, \theta_i = \arctan(v_i/u_i)$$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